

Ecuación para el diseño de reactores de electrocoagulación con flujo vertical ascendente

J.J. Canto-Ríos^a, A. Martín-Domínguez^b, C. Vales-Pinzón^{a,*},
H.A. Pacheco-Martínez^a, R. González-Herrera^a

^aFacultad de Ingeniería de la Universidad Autónoma de Yucatán

^bMexican Institute of Water Technology, Paseo Cuauhnáhuac 8532, Progreso, Jiutepec, Morelos, México

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Resumen

En este trabajo, se comparan los resultados experimentales de un reactor de electrocoagulación de flujo vertical con las predicciones de una ecuación de diseño propuesta. Para generar el espacio muestral y examinar el impacto de las variables tales como el número de placas (N_p), la densidad de corriente (j), el número de módulos (N_m) y la separación (s) de electrodos en los resultados de la ecuación de diseño, se utilizó el modelo experimental 2^k . En este estudio, hemos enfocado nuestro análisis en la densidad de corriente eléctrica, que definimos como la relación entre la intensidad de corriente y el área de superficie de la placa, así como en el número de placas, con el objetivo de determinar su influencia en la ecuación de diseño. Nuestro análisis experimental reveló que estos factores son los más sensibles en la ecuación de diseño. De acuerdo con los resultados, el impacto de la densidad de corriente aumenta en el rango de 2 a 10 placas, después de eso se vuelve constante, por otro lado, la relación entre la densidad de corriente y el número de placas muestra la importancia del análisis de sensibilidad. Finalmente, el modelo ANOVA se ajusta con el 95% con el consumo de voltaje calculado.

Palabras Clave: Corriente, electrocoagulación, diseño experimental factorial, electrodos, flujo de fluidos.

Equation for the design of electrocoagulation reactors with vertical ascending flow

Abstract

In this work, the experimental results for a vertical flow electrocoagulation reactor are compared with the predictions of a proposed design equation. The 2^k experimental model was used to generate the sample space to examine the impact of the variables Number of plates (N_p), current density (j), Number of modules (N_m), and electrode separation (s) on the results of the design equation. In this study, the electric current density which is defined as the ratio between current intensity and the plate surface area, and the number of plates were analyzed to determine their influence on the design equation. The experimental analysis revealed that these factors are the more sensitive in the equation. According to the results, the current density impact increases in the range of 2 to 10 plates, after that it becomes constant, on the other hand, the relation between the current density and the number of plates shows the importance of the sensibility analysis. Finally, the ANOVA model fits with 95% with the voltage consumption predicted.

Keywords: Current, electrocoagulation, factorial experimental design, electrodes, fluid flow.

*caridad.vales@correo.uady.mx

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1 Introduction

Coagulation is the chemical destabilization of colloidal particles caused by adding chemical coagulants and applying energy which neutralizes the forces that keep the particles from adhering to each other [1].

Electrocoagulation (EC) is an emergent alternative proposal to chemical coagulation (CQ). Three mechanisms interact in electrocoagulation influencing its efficiency: hydraulic, electrochemical, and physicochemical (Figure 1). Some of the factors influencing the EC efficiency through the mechanisms mentioned above are listed in Table 1.

The physicochemical and electrochemical aspects have been widely discussed in the literature. However, a methodology to design an electrocoagulation reactor has not been reported to the best of our knowledge [2]; the relation that exists among operation variables in Table 1 which there are plenty of questions and the hydrodynamic of the system that is necessary to clear to optimize the design in this kind of reactors.

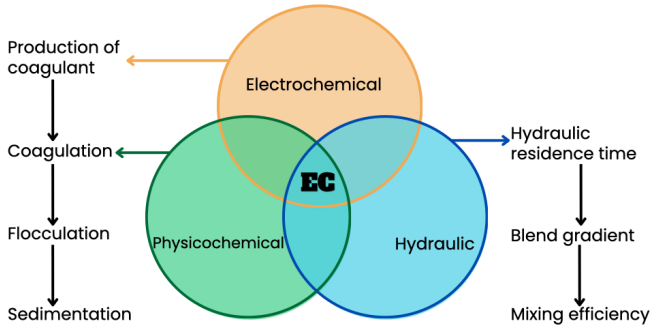


Figure 1: Mechanisms that combine during electrocoagulation.

Hakizimana [3] mentions that one of the weaknesses of this methodology is the absence of understanding of

the interaction from the different processes presented. It is necessary to assess comprehensively the geometric and operational variables present in the electrochemical phenomenon, in a way that allows analysis of the design parameters' influence (electrical current density, electrode separation, number of electrodes, number of modules), and how they are related to operation variables and the hydraulic system, mainly voltage drop and hydraulic head losses.

This work shows a significant advance in the understanding and design of vertical flow electrocoagulation reactors. Although the physicochemical and electrochemical aspects of electrocoagulation have been widely discussed in the literature, there is a notable lack of methodologies to effectively design these reactors, especially regarding the relationship between the operational variables and the hydrodynamics of the system. The research addresses the knowledge gap through the proposal of an equation that predicts either the potential or voltage in an electrocoagulation reactor with a free surface and vertical ascending flow (ascendant/descendent), based on design and operational parameters in this type of reactor. The prediction of voltage is important because it would help in choosing the operational range for power sources and in estimating the energy consumption during operation. This equation represents a crucial step toward the design optimization of these reactors, providing a tool for selecting the operating range of energy sources and for estimating energy consumption.

Additionally, through a 2^k model and variance analysis, this study delves into the influence of critical factors such as current density and the number of plates, demonstrating the importance of sensitivity analysis in the design equation. The analysis of the influence of involved parameters on the model's accuracy compares the obtained values under real conditions, using variance analysis methodology through a design of experiments.

Table 1: Involved factors in the EC efficiency.

Physicochemical	Electrochemical	Hydrodynamics
Conductivity	Current density	Blend gradient
pH	Electrode material	Flow velocity
Pollutant type	Polarity change	Hydraulic residence time (TRH)
	Distance between electrodes	Flow type (laminar or turbulent)
	Electric connection type	Operation type (pressure or gravity)

2 Design parameters

Different studies have been carried out about this technology and these works have been focused mainly on validating and reviewing the advantages of electrocoagulation as an alternative to chemical coagulation [4–8]. On the other hand, they have reviewed the variation

from different design parameters, to optimize the process. From the above mentioned, it has been observed that those who have the biggest effect on the electrocoagulation efficiency process during the pollutant removal are: electrical current density, the coagulant dose, and the number of separations of the electrodes.

The difference of the potential or voltage Z is a variable that affects the operation cost directly, is defined as the necessary work to transfer a load unit, and depends mainly on the solution conductivity, applied electric current, electrode separation, electrode total area and in general the resistance that opposes the system when the current goes through.

2.1 Current Density

By Ghernaout et al [9], the electric current is the foremost parameter from EC that affects the current density directly. The current density j is the amount of electric current I , amperes (A) or coulombs per second (C/s), applied per unit area of electrode anode (A_r) per square meter (m^2):

$$j = \frac{I}{A_r} \quad (1)$$

To high densities, the electrodes' passivation probabilities increase. Nevertheless, when they have low current densities, it is required to raise the electrode areas, impacting the reactor volume, at the residence time, thus, in the size of the units and investment cost [10,11].

Gelover et al. [12] have carried out studies about the impact of current density in silica removal, using a reactor that had an array of electrodes in parallel, which forced the flow either to ascent or descent movement through them. They found that is possible to have high treatment efficiencies with low current densities. However, achieving this fact requires a high number of plates which increases the size of the reactor, and a greater amount of electric connections.

Hakizimana et al. [3] present a review of the development and design of the EC process. They mention that the geometry, the feeding type, and the mixing conditions affect the current density distribution.

2.2 Coagulant Doses

The required coagulant doses depend on the pollutant concentration that it is desired to remove but, the amount generated electrochemically (C_c) in g/L in situ is a function of Faraday law [13,14]. It is shown that C_c is directly proportional to the current I that goes through the system as well as at the time of application of this (t), in seconds (s) per unit volume (V) in cubic meter (m^3).

$$C_c = \frac{ItPM}{VnF} \quad (2)$$

where n is the number of electrons transferred to moles of the transformed reagent (coagulant), F is the Faraday constant (96,500 C/mol), and PM (g/mol) is the molecular weight.

2.3 Electrode spacing and configuration

The electrode spacing affects the efficiency and cost of electrochemical processes. By Sahu et al. [8], during the electrolysis, the solution in the vicinity of the anode has greater coagulant concentration and a high pH, due to the boundary layer influence. Khandegar et al. [7] indicate that the distance of the electrodes plays a significant role in the electrocoagulation, due to the electrostatic field depending on the distance between the anode and cathode. They found that for a small distance, the removal efficiency is low, due to some flocs getting broken by the collision that generates the electrostatic attraction and the shear stress between the plates. However, the endurance to the passage of current rises with the separation of the electrodes which elevates the voltage requirements as well as the operating costs. On the other hand, the volume between electrodes fills up with gases partially during the electrolysis, increasing as well as the electric resistance. The vertical position of the electrodes and their geometry play an important role in the gas release; due to that, the open settings with vertical electrodes have proven to have better effectiveness. Moreover, it promotes the flotation of pollutants and the precipitation of the generated sludge [15]. Not only, the electrical consumption is reduced through the distance reaction between electrodes, but also it generates a greater amount of tiny gas bubbles due to the hydrodynamic turbulence [9]. Additionally, Brahmi et al. [16] analyzed several distances between electrodes in the interval of 0.5 and 2 cm. When the distance between electrodes increases as well as the endurance of the energy transfer. Therefore, the load transfer kinetics and the aluminum oxidation are reduced.

2.4 Number of electrodes

Chen et al. (2004) mention that the electrode number to use should be between 10 and 100 [17]. On the other hand, Piña-Soberanis et al. describe that few authors report information about the distance, size, or active area of the electrodes, and as has been mentioned before, they are important parameters for design [6]. The number and orientation of the electrodes depend on the reactor settings and the flow type that is desired in the system. Mainly, it seeks to increase the contact surface with the water to treat and decrease the hydraulic residence time, but also, influences the number of electrodes as well as the size in the current density, hence, in the passivation of them. The reactor settings by modules in series, the distribution of the electrodes helps the flow mixing throughout the reactor but, it implies that each module produces a coagulant fraction desired, which means, the required doses will be obtained at the end of the last module according to the reactor.

2.5 Reactor Settings

Villegas mentions that when the pressure in reactors with piston flow, the mixing energy is dissipated due to direction changes, reductions, and extensions of sections as well as by the fraction of straight sections formed by the electrodes [18]. Nevertheless, the head loss mentioned in its experimentation was mainly caused by the presence of deposits on the electrode's surface. Gelover *et al.* worked with a rectangular cross-section reactor, with electrodes separated 0.6 cm and horizontal maze-type flow formed by the plates [19]. This geometry allowed for to reduction of death regions and short circuits from the hydraulic point of view. Also, they tested for silica removal in electrocoagulation closed reactors with vertical flow; they observed that the silica removal was greater with high velocities due to the dragging of deposits formed on the electrodes. Later, Gelover *et al.* compared either open or closed reactor settings for water conditioning in the industry, and they concluded that the open setting allows them to reach the same removal efficiencies as the closed reactors [20]. It represents such an advance in the development of this technology. However, there are still questions about the combined effect of the design parameters in the electrocoagulation reactors. Due to reactor efficiency with open vertical flow proved by Gelover *et al.* [19,20], the present research is focused on the hydraulic model for design from this sort of reactor.

3 Methods

3.1 Study model

To propose the design equation is considered that each electrocoagulation treatment unit is a compound of several finite modules N_m , where are located the electrodes (plates), shown up in Figure 2. Each one of these models is compounded by two regions: declination (1) and reaction (2), the last one where the electrodes are placed. The geometry from both zones and the communication orifice between them (drowned orifice) is related to the used electrodes N_p and the separation s between them; which allows keeping the same speed in all the flow paths. Figure 2 shows that the distance hp is used to homogenize the ascendant streamlines to the reaction region and decrease death regions and short circuits. From the model, the equations are combined, in such a way that the geometry of the unit is related to the hydraulic and electrochemical operation.

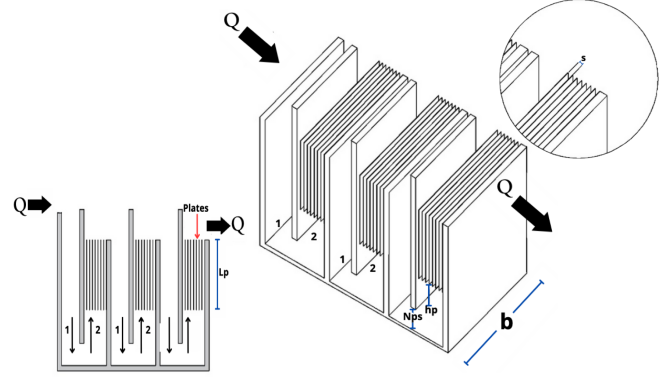


Figure 2: Isometric scheme of the electrocoagulation reactor used in this research (on the left) with three modules. To the right is a longitudinal cut.

3.2 Coupling Equations

In an electrocoagulation system, the first step is to set up the amount of required coagulant, as a function of the pollutant to remove and from its concentration in the water. Hence, considering the Faraday law (Equation 2) is cleared the electric current, obtaining Equation 3.

$$I = \frac{C_c V n F}{P M t} \quad (3)$$

The previous expression for reactors in Batch, where the current depends on the desired coagulant concentration and of the study volume. For continuous flow is used the caudal definition, $Q = \text{volume/time}$, and Equation 3 is rewritten as:

$$I = \frac{C_c Q n F}{P M} \quad (4)$$

To calculate the flow in the system, it is considered that the discharge is from the inferior orifice to the next tank whose level of water is above the orifice. It means a drowned discharge, which is used in Equation 5, where the discharge coefficient could take values of a drowned orifice [21]. This equation depends also on the orifice cross-section area (A_f) and the square root of twice the gravity acceleration (g in m/s^2) multiplied by the difference of levels between the containers (ΔH in meters).

$$Q = C_d A_f \sqrt{2g\Delta H} \quad (5)$$

The surface of the cross-section to the flow results in the product of separation (s in meters) from the plates, the number of plates N_p (when one of the end plates is attached to the wall preventing the flow), and for the wide of the reactor (b in meters). Therefore, the equation for A_f becomes:

$$A_f = b N_p s \quad (6)$$

Using Equations 4 to 6 where C_c is the dependent variable, the expression in the function of supply current, the geometry of the reactor, and the loss of Hydraulic head is obtained:

$$C_c = \frac{IPM}{nFC_d b N_p s \sqrt{2g\Delta H}} \quad (7)$$

In the current density definition (see 1), A_r is the reaction area, which means that the electrode total area corresponding to the anode is calculated as:

$$A_r = bL_p(N_p - 1)N_m \quad (8)$$

where L_p is the plate height (in meters) and N_m is the number of modules according to the reactor. Substituting Equations 1 and 8 in 7:

$$C_c = \frac{jPML_p(N_p - 1)N_m}{nFC_d N_p s \sqrt{2g\Delta H}} \quad (9)$$

ΔH corresponds to a loss of energy in the reactor, and it could be understood as the difference between the inlet load of the reactor H_T and the weir load to the outlet of the reactor H_v :

$$\Delta H = H_T - H_v \quad (10)$$

It is considered that all the modules are the same. Hence, they have the same number of plates in each of them. In addition, it is noteworthy that due to the type

of reactor studied, the modules operate in series. Therefore, all the flow goes through each one of them. The voltage drop through the electrolyte (V_{elec} in Volt) is determinate following the Ohm law [22], by the applied electric charge (I in ampere), the anode surface A_r , the distance s between the anode and cathode, and the electric conductivity of the electrolyte γ_{H_2O} (in Siemens/m).

$$V_{elec} = I \frac{s}{\gamma_{H_2O} A_r} = j \frac{s}{\gamma_{H_2O}} \quad (11)$$

For this voltage, it is necessary to add what is required to beat the electric connection resistance (V_{con}). Hence, the voltage to apply to the system V_{sis} is:

$$V_{sis} = V_{elec} + V_{con} \quad (12)$$

The voltage drop is applied using Ohm's law, considering all the elements that represent resistance to the current, and is multiplied by the current that passes through them [23]. For the electric connections V_{con} is considered the material strength R_c (in ohm), and the current in the function of the number of electrodes (N_p), the operation current density j and the length of the cable L_c .

$$V_{con} = R_c L_c 2j(N_p - 1)bL_p \quad (13)$$

When Equations 9 and 13 are combined and V_{sis} is cleared, is obtained the next relation:

$$V_{sis} = \frac{j^2 P M L_p (N_p - 1) N_m}{n F C_d N_p \sqrt{2g\Delta H} C_C \gamma_{H_2O}} + 2j(N_p - 1)bL_p R_c L_c \quad (14)$$

To this voltage value could be added the initial system (V_{FP}). However, that varies in each case and the equation would lose generality [22]. Once the equation is defined, is necessary to review the fundamental di-

mensions of each variable since this helps to verify the veracity of physics formulas, using the principle of dimensional homogeneity, Table 2 shows them.

where: [L] = length, [M] = mass, [T] = time, [I] = current intensity and [N] = amount of substance in mols.

Substituting the units from the chart that is expressed at Equation 14 and simplifying, is obtained:

$$V_{sis} = [M][L]^2[T]^{-3}[I]^{-1}$$

The design equation is homogeneous. Nevertheless, it depends on four variables, is necessary to solve it through iterative methods under minimization conditions. Therefore, Equation 14 that defines the system voltage was evaluated and partially derived for the different design parameters, allowing us to know the change that occurs in the voltage when modifying each one of

these.

In Equation 14, the component corresponding to the voltage required by the system is reviewed. For this, the sensitivity analysis was carried out under a Monte Carlo simulation approach, using 1000 simulations, with the parameters individually, and later with the possible combinations between variables. A uniform distribution was considered, this reactor design equation was applied and subsequently, the sensitivity coefficient was calculated for each of the parameters of the model [24] according to the equations described below:

The relative-absolute sensitivity describes the relative change of the result due to a relative change in the

Table 2: Fundamental dimensions of the variables.

Magnitude	Symbol	Dimension	S.I.
Current	I	$[I]$	$A=C\ s^{-1}$
Faraday constant	F	$[I][T][N^{-1}]$	$C\ mol^{-1}=A\ s\ mol^{-1}$
Current density	j	$[I][L]^{-2}$	$C\ s^{-1}m^{-2}=A\ m^{-2}$
Hydraulic head	H	$[L]$	m
Coagulant concentration	C_c	$[M][L]^{-3}$	$g\ m^{-3}$
Anode area	A_r	$[L]^2$	m^2
Cross section to flow	A_f	$[L]^2$	m^2
Gravitational acceleration	g	$[L][T]^{-2}$	$m\ s^{-2}$
Water conductivity	γ_{H_2O}	$[I]^2[T]^3[L]^{-3}[M]^{-1}$	$S\ m^{-1}$
Connector wire resistivity	R_c	$[M][L][I]^{-2}[T]^{-3}$	ohm/m
Electric potential	V_{sis}	$[M][L]^2[I]^{-1}[T]^{-3}$	V
Molecular weight	PM	$[M][N]^{-1}$	g/mol
Plates lengths	L_p	$[L]$	m
Conductor material length	L_c	$[L]$	m

parameter.

$$s_{i,j}(\theta_{M,j}) = \frac{1}{f(\theta_{M,j})} \frac{\partial f(\theta_{M,j})}{\partial \theta_{M,j}} \quad (15)$$

Absolute-relative sensitivity describes the relative change of the result due to an absolute change in the parameter.

$$s_{i,j}(\theta_{M,j}) = \theta_{M,j} \frac{\partial f(\theta_{M,j})}{\partial \theta_{M,j}} \quad (16)$$

At last, the relative-relative sensitivity equation is used to describe the relative change of the results with the relative change of a parameter at 100%.

$$s_{i,j}(\theta_{M,j}) = \frac{\theta_{M,j}}{f(\theta_{M,j})} \frac{\partial f(\theta_{M,j})}{\partial \theta_{M,j}} \quad (17)$$

where $f(\theta_{M,j})$ represents the n output variables, $\theta_{M,j}$ for the independent parameter m and $\partial \theta_{M,j}$ is the range for the parameter variability j .

3.3 Design of experiments

The design of an electrocoagulation reactor of the free surface, with biphasic vertical flow, might be done considering simultaneously the electric and hydraulic phenomenon through the proposal design Equation 14. This equation allows us to calculate voltage as a function of the main four parameters: current density j , plates separation s , number of plates N_p , and number of modules N_m . Voltage is important in the design of those systems because it influences the energy costs directly thus to minimize it, testing with the dimensional characteristics and functioning that it would allow to optimize the operation cost.

To analyze which variables had a significant influence on the accuracy of the proposal design equation, a factorial experimental design 2^k was used. This enables us to

obtain the voltage difference ΔV between the measured V_R in the electrocoagulation reactor described in Figure 2 and the voltage was predicted by Equation 14 (V_{sis}). With this design the study variables k on two levels are varied, and it is analyzed the effect them as well as their iterations, on ΔV . The analysis of variance (ANOVA) allows us to do a filtering on the variables that affect a significant statistic way, with 95% confidence, to the studied response.

The experiments have been carried out using a module from the described reactor in Figure 2, starting with the consideration that all the modules would share in the same way, and the produced aluminum concentration in each module should be the same.

The electric connection of the electrodes was in a parallel way, monopole and galvanic static operation, which means, it is placed to a current intensity and is registered either a potential or voltage. The plates were made of aluminum with commercial dimensions such as $1.5\ m \times 0.09\ m \times 0.00123\ m$, which correspond to length, width, and thickness, respectively. The water used during the experiment was taken directly from the tap, and sodium chloride was added to it in a container before being introduced into the reactor to maintain a constant conductivity of $750\ \mu S/cm$. The total theoretical concentration from produced aluminum, independently from the number of modules, is set out at $26.9\ mg/L$. Not only water conductivity but also aluminum concentration is focused arbitrarily, just to get reference values given to in this goal research was not water treatment instead, it is the analysis from reactor design equation.

Once the experimental voltages had been obtained, an iteration matrix was built as well as the responses of each combination through analysis of variance, to meet those variables that were significant for the analyzed response (ΔV).

4 Results and discussion

4.1 Voltage drop analysis

The actual voltages obtained in each one of the experiments are presented in column V_R , whilst the technical values on V_{sis} at Table 3; the response variable of the design of experiments is an absolute value from the dif-

ference between those two variables and they are indicated in column $|\Delta V|$. In each experiment displayed in Table 3 the variables considered different values, based on the fact that these types of techniques studied the effects of all factors of interest simultaneously. At the same time, to establish the factor values that should be modified corresponding hydraulic and electrochemical parameters, they are displayed in Table 4.

Table 3: Voltage results obtained from the design of experiments.

Exp. #	j (A/m ²)	s (m)	N_p	N_m	V_R (V)	V_{sis} (V)	$ \Delta V $ (V)
1	20	0.006	3	4	3.63	1.68	1.94
2	60	0.006	3	4	5.1	5.50	0.36
3	20	0.015	3	4	5.7	3.38	2.32
4	60	0.015	3	4	14.3	12.73	1.57
5	20	0.006	8	4	4.4	2.11	2.29
6	60	0.006	8	4	9.45	6.06	3.39
7	20	0.015	8	4	3.9	3.93	0.03
8	60	0.015	8	4	12.1	14.25	2.16
9	20	0.006	3	6	5.4	1.785	3.62
10	60	0.006	3	6	5.1	5.44	0.34
11	20	0.015	3	6	5.3	4.24	1.06
12	60	0.015	3	6	15.5	13.48	2.01
13	20	0.006	8	6	4.28	2.02	2.26
14	60	0.006	8	6	12.1	6.09	6.01
15	20	0.015	8	6	3.75	4.15	0.40
16	60	0.015	8	6	17.4	14.01	3.39

Table 4: Hydraulic and electrochemical parameters during the experiments.

Exp. #	Flow (L/min)	Water velocity between plates (m/s)	Plates surface per module (m ²)	Current per module (A)	Theory aluminium per module (mg/L)	Real aluminium per module (mg/L)
1	4.48	0.046	0.270	5.40	6.74	8.4
2	13.44	0.128	0.270	16.20	6.74	13.6
3	4.48	0.018	0.270	5.40	6.74	10.5
4	13.44	0.055	0.270	16.20	6.74	12.0
5	15.68	0.061	0.945	18.90	6.74	7.2
6	47.05	0.182	0.945	56.70	6.74	9.7
7	15.68	0.024	0.945	18.90	6.74	7.3
8	47.05	0.073	0.945	56.70	6.74	10.1
9	6.72	0.069	0.270	5.40	4.50	11.5
10	20.17	0.207	0.270	16.20	4.50	7.8
11	6.72	0.028	0.270	5.40	4.50	5.1
12	20.17	0.083	0.270	16.20	4.50	7.8
13	23.53	0.091	0.945	18.90	4.50	6.1
14	70.58	0.272	0.945	56.70	4.50	4.8
15	23.53	0.036	0.945	18.90	4.50	6.7
16	70.58	0.109	0.945	56.70	4.50	6.5

Figure 3 must be observed that the results for (V_{sis}) are similar to voltage values found out in the experimentation (V_R). The minimum difference found was 0.03 and the maximum 6.0 volts, getting an average of 2.07 volts.

An analysis of variance (ANOVA) was run from several factors (Table 5), with decoded values, to determine which of them have any significant statistical effect on the difference between theories and real voltages, through the F test. Also, the iteration's significance was evaluated and the generalized linear model was obtained that was the best in the data adjustment. The statistician R^2 indicates that the model explains 94.47% of the variability in the response. The statistician adjusted R^2 , which is the most appropriate to compare models with different numbers of independent variables, is 72.36%. Since the Value-P is higher than 0.05, there is no indication of a serial autocorrelation in the remainder with a confidence level of 95.0%. On the other hand, the Nash-

Sutcliffe coefficient indicates with a 69% that the model describes very well the voltage consumption.

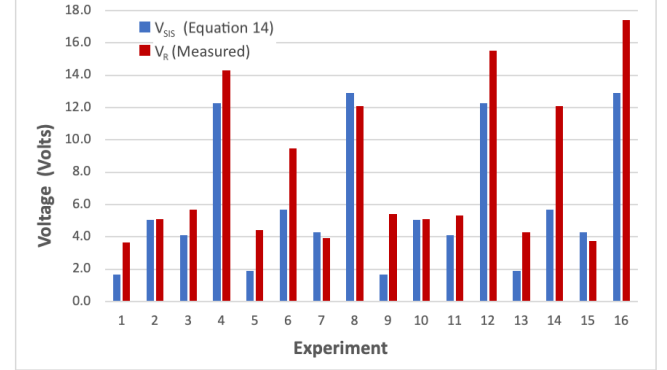


Figure 3: Voltage comparison measured in the laboratory with calculated data using the proposal design equation.

Table 5: Analysis of variance.

Parameter	Sum of square	Degrees of freedom	Middle Square	R^2	Adjust R^2	Values-P
Model	33.70	12	2.81	94.47	72.36	0.1290
Residual	1.97	3	0.66			
Total	35.67	15	2			

Table 6 displays the results of fitting the multiple linear regression model to describe the relation between response (voltage difference) and 12 independent variables. Also, the statistician T is presented, which is compared with the critical value obtained from the charts through degrees of freedom from the remainder, in that case, the value is 3.18; which indicates that the single variable has a significant influence on the response variable, with a 95% of confidence, is the iteration $j * N_p$, due to it was the only one which value is $T > 3.18$.

Table 6: Estimated values from the lineal and T statistic model.

Parameter	Estimation	T-Statistic
Constant	2.07114610.22	
j	0.33132	1.63
s	-0.45469	2.24
N_p	0.41928	2.07
N_m	0.31498	1.55
js	0.33261	1.64
jN_p	0.91310	4.51
jN_m	0.22026	1.09
sN_p	-0.54092	2.67
sN_m	-0.21686	1.07
N_pN_m	0.20948	1.03
jsN_p	-0.30135	1.49
jN_pN_m	0.21815	1.08

The equation of the adjusted linear model is shown in Equation 18, according to the results given by the ANOVA analysis.

$$\begin{aligned}
 \Delta V = & 2.07114 + 0.33132j - 0.45469s \\
 & + 0.41928N_p + 0.31498N_m + 0.33261js \\
 & + 0.91310jN_p + 0.22026jN_m - 0.54092sN_p \\
 & - 0.21686sN_m + 0.20948N_pN_m - 0.30135jsN_p \\
 & + 0.21815jN_pN_m
 \end{aligned} \quad (18)$$

Figure 4 corresponds to the linear model response obtained from the design of experiments, the fit is in agreement between the theoretical and experimental results.

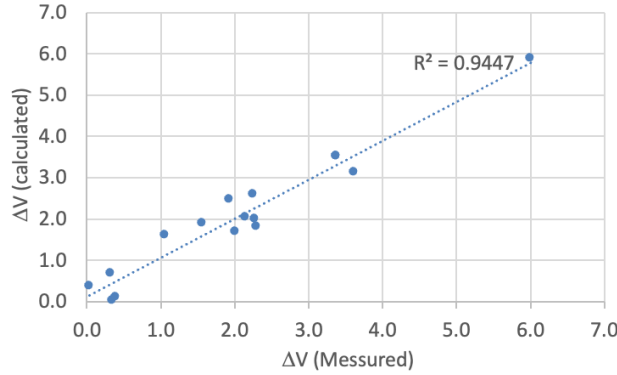


Figure 4: Graphic of the adjusted linear model, obtained from the design of experiments.

4.2 Variables sensitivity analysis

The values that minimize the voltage difference using the adjusted linear model, are the following: j its encoded value (+1) corresponds to 60 A/m², and the other three variables, their encoded values (-1) corresponds to s in 0.006 m, N_p on 3 plates and N_m on 4 modules. However, there is a significant interaction effect between j and N_p , values from ΔV are increasing if those two variables are at their highest value as well as if they are at the lowest value simultaneously. It means, low density with few plates in the module. Hence, it generates more errors in the theoretical calculation of voltage. Figure 5 represents the surface response graphic of the adjusted linear model, focusing s its encoded value (-1) and N_m its encoded value (-1).

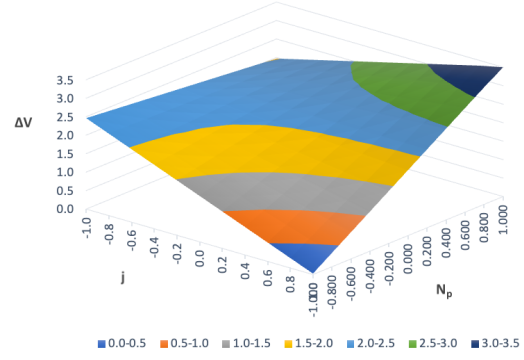


Figure 5: Surface graphic of the adjusted lineal model, focusing in s (-1) y N_m in (-1).

Figure 6 shows the behavior of the voltage rate with respect to the current according to the number of plates and modules, keeping the current density constant. It is shown that the increase when modifying the number of modules obeys a linear behavior, while the increase in the number of plates has a greater change in voltage in the first values, later the voltage variation stabilizes in small variations.

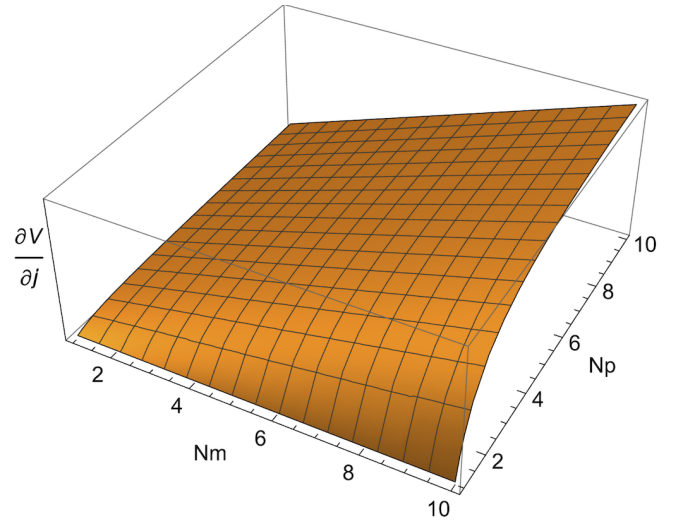


Figure 6: Voltage rate with respect to the current according to the number of plates, and modules.

In Figure 7 the behavior of the voltage rate for the number of modules is shown, as a function of current and number of plates. The current density presents an ascending parabolic type, while the number of plates keeps the same behavior of tendency to reduce the voltage variation with the increase in the number of plates, which indicates a rate of change at a higher rate due to the current density.

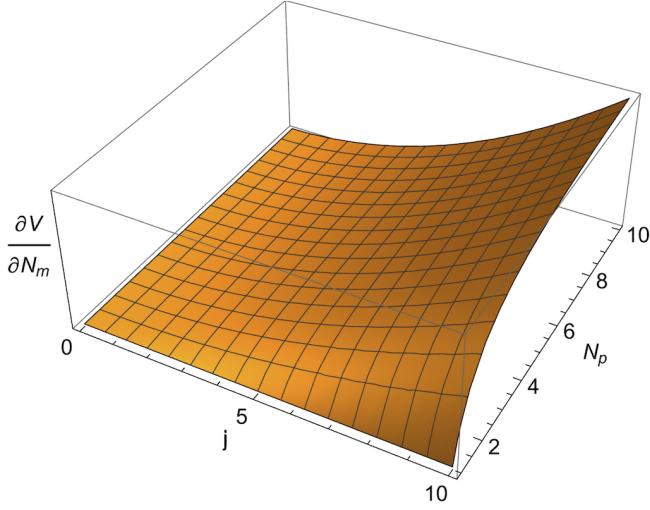


Figure 7: Voltage rate with respect to the number of modules according to the current density, and number of plates.

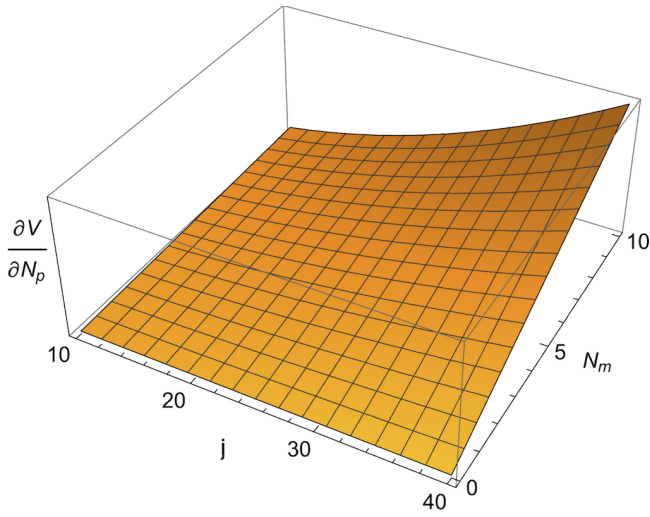


Figure 8: Voltage rate with respect to the number of plates concerning the number of modules, and the current density.

By keeping the number of plates constant, it is observed that the voltage rate in this parameter has an ascending parabolic type behavior considering the variation in current density, and it is more evident the more modules there are. On the other hand, it shows an ascending linear behavior due to the variation in the number of modules (See Figure 8).

Figure 9 presents the dependence on the variables j , N_p , and N_m of the voltage and its partial change rate from the point of each of them. In the case of current density, a linear variation would be expected, having a significant growth impact on the sizing of the system.

The impact of the number of modules results in lin-

earity, favoring system scaling. The number of plates has a significant effect on systems with a small number of them, however, there is a limit on said parameter such that the voltage will remain stable with increases in the number of plates.

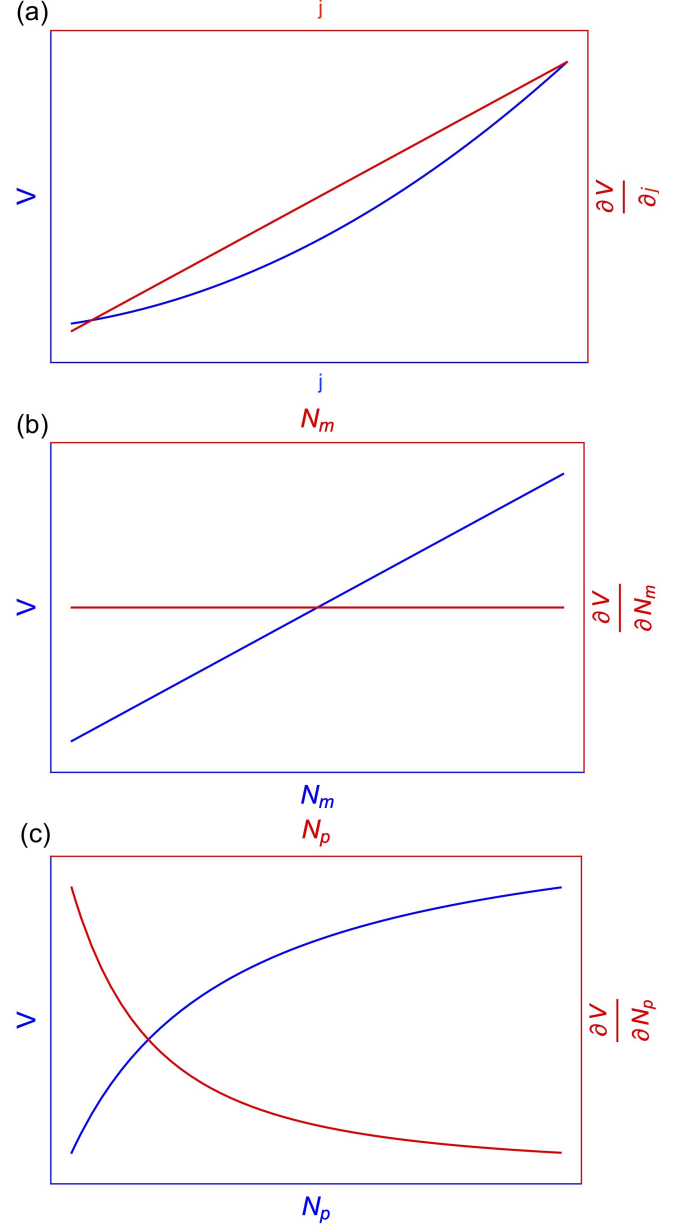


Figure 9: Voltage gradient behavior a) changing the current density b) changing the number of modules and c) changing the number of plates.

The results of the sensitivity analysis performed on the design equation using Equations 15 to 17, allow us to determine the relationship between each of the parameters involved. Figure 10 shows the sensitivity index of each of the parameters used for the design. Analyzing the individual impact of the variables, it is observed

that the sensitivity is reduced by an increase in current density or the number of modules. In such a way, having more than a dozen modules ceases to be significant. On the other hand, the behavior of the variation of the Voltage with respect to the number of plates shows a rapid variation when modifying them, showing high sensitivity with a low number of these. However, as the plates increase, the sensitivity is rapidly reduced and remains constant. On the other hand, if the interactions between two variables were considered, the one with the most significant impact is the current density with the number of modules, maintaining the number of fixed plates. Followed by the current density with the number of plates and the one with the least impact is the number of modules with the number of plates, keeping the third parameter constant.

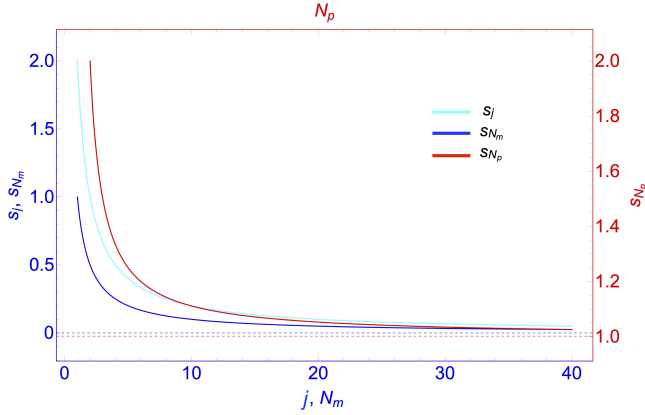


Figure 10: Sensitivity analysis for the design variables.

The relative-absolute sensitivity presented in the interactions of the variables, the most important the density of current with the number of modules, followed by the density of current, the number of modules, and the number of plates, being the smallest the number of plates with the number of modules as shown in Figure 11.

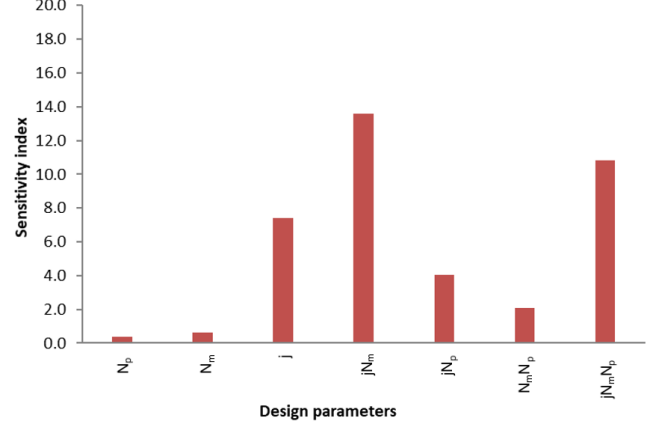


Figure 11: Relative-absolute sensitivity index for the design Equation 15.

Like the previous index, for absolute-relative sensitivity one it is observed that the sensitivity is presented in the interactions of the variables, being the most important the density of current with the number of modules, followed by the density of current and the number of plates, being the smallest the number of plates with the number of modules, as shown in Figure 12.

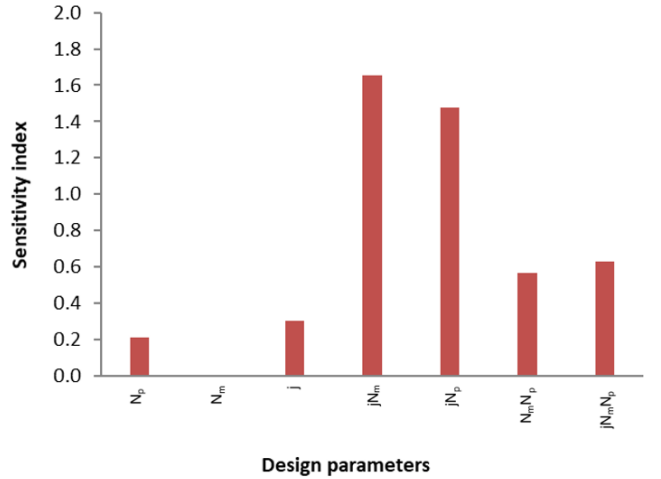


Figure 12: Absolute - relative sensitivity index for the design Equation 16.

Finally, the relative-relative index presents a greater sensitivity when the interaction of two variables is presented, in this case, the one with the highest sensitivity is the interaction of the current density with the number of modules, followed by the current density and the number of plates, as shown in Figure 13.

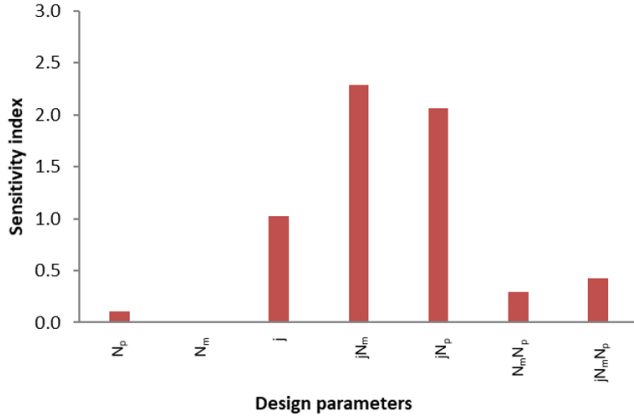


Figure 13: Relative-relative sensitivity index for the design Equation 17.

When comparing the results obtained with the sensitivity indices and graphs of the equation, it is observed that the behavior is coherent. The indices show that the relationship of the current density with the number of modules is greater than the interaction of the current density with the number of plates when compared with the corresponding graph. It is observed that the change of voltage is constant and continuous with the number of modules, while Figure 10 shows when increasing the number of plates, the voltage increase is smaller and tends to be constant.

5 Conclusions

Experiments were designed to analyze the significant influence of the parameters involved in the design equation of an electrocoagulation reactor of vertical flow with multiple modules.

It allowed establishing that current density interaction and the number of plates ($j * N_p$) are the factors that most influence the accuracy of voltage calculation needed to produce a determinate coagulant concentration. The slightest mistake between theoretical and real voltage is given when the current density is high and there are few plates in the reactor, less than 10 plates. It could mean that in the proposal design equation, a term is missing that considers the ohm drop generated when the number of plates and density is increasing, as well as when those factors are decreasing, on the other hand, that would indicate that the importance of the plates is only with a smaller number of it, after ten plates, the increment is not significant. The equation has three main parts, the residual voltage of the source of power, the voltage of the reactor, and the drop of energy of the wire, the only one that is independent of the reactor design is the first one. Due to this is important to analyse how this value changes in each case. The resulting model has an accuracy of 95%, meanwhile, the voltage consumption is well described with a 70% accuracy. The main portion of the drop is given by the plate system, which is important to study the change caused by the reaction in the plate's separation.

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